Subspaces and Bases

Subspace

DEFINITION

A non-empty subset $V \subseteq \mathbb{R}^n$ is called a ***subspace*** if for all $\vec{u}, \vec{v} \in V$ and all scalars k we have

(i) $\vec{u} + \vec{v} \in V$; and

(ii) $k\vec{u} \in V$.

Subspaces give a mathematically precise definition of a "flat space through the origin."

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34 For each set, draw it and explain whether or not it is a subspace of $\mathbb{R}^2$.

34.1 $A = \left\{ \vec{x} \in \mathbb{R}^2 \,:\, \vec{x} = \begin{bmatrix} a \\ 0 \end{bmatrix} \text{ for some } a \in \mathbb{Z} \right\}$.

34.2 $B = \left\{ \vec{x} \in \mathbb{R}^2 \,:\, \vec{x} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\}$.

34.3 $C = \left\{ \vec{x} \in \mathbb{R}^2 \,:\, \vec{x} = \begin{bmatrix} 0 \\ t \end{bmatrix} \text{ for some } t \in \mathbb{R} \right\}$.

34.4 $D = \left\{ \vec{x} \in \mathbb{R}^2 \,:\, \vec{x} = \begin{bmatrix} 0 \\ t \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ for some } t \in \mathbb{R} \right\}$.

34.5 $E = \left\{ \vec{x} \in \mathbb{R}^2 \,:\, \vec{x} = \begin{bmatrix} 0 \\ t \end{bmatrix} \text{ or } \vec{x} = \begin{bmatrix} t \\ 0 \end{bmatrix} \text{ for some } t \in \mathbb{R} \right\}$.

34.6 $F = \left\{ \vec{x} \in \mathbb{R}^2 \,:\, \vec{x} = t\begin{bmatrix} 3 \\ 1 \end{bmatrix} \text{ for some } t \in \mathbb{R} \right\}$.

34.7 $G = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$.

34.8 $H = \operatorname{span}\{\vec{u}, \vec{v}\}$ for some unknown vectors $\vec{u}, \vec{v} \in \mathbb{R}^2$.